**Chapter 4: Rational Functions and Conics**

4.1 – Rational Functions and Asymptotes

A **rational function** can be written in the form:

$$f\left(x\right)=\frac{p(x)}{q(x)}$$

where $p\left(x\right)$ and $q(x)$ are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and $q(x)$ is not the \_\_\_\_\_\_\_\_\_

polynomial.

In this section, we assume that $p\left(x\right)$ and $q(x)$ have no \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

In general, the *domain* of a rational function of $x$ includes \_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

except $x-values$ that make \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Much of our discussion of rational functions will focus on their graphical behavior near these $x-values. $

**FINDING DOMAIN OF A RATIONAL FUNCTION**

Find the domain of $f\left(x\right)$ and discuss the behavior of $f$ near any excluded $x-values.$

 $f\left(x\right)=\frac{1}{x}$

To determine the end behavior of $f$ near this excluded value, evaluate $f(x)$ to the left and right of $x=$ \_\_\_\_\_\_.

**To the left of \_\_\_\_\_**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| $$x$$ | $$-1$$ | $$-0.5$$ | $$-0.1$$ | $$-0.01$$ | $$-0.001$$ | $$\rightarrow 0$$ |
| $$f(x)$$ |  |  |  |  |  |  |

**To the right of \_\_\_\_\_**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| $$x$$ | $$0\leftarrow $$ | $$0.001$$ | $$0.01$$ | $$0.1$$ | $$0.5$$ | $$1$$ |
| $$f(x)$$ |  |  |  |  |  |  |

As $x$ approaches 0 *from the left*, $f(x)$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ without bound, whereas as $x$ approaches 0 *from the right,* $f\left(x\right)$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ without bound.

THE GRAPH OF $f\left(x\right)=\frac{1}{x}$

The line \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of the graph $f$.

The graph of $f$ also has a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ – the line \_\_\_\_\_\_\_\_\_.

Examples: Find the domain of the following rational functions.

1. $f\left(x\right)=\frac{1}{x+1}$ b. $f\left(x\right)=\frac{3x^{2}}{x^{2}-9}$ c. $f\left(x\right)=\frac{5x}{x^{2}+3x-10}$



d. e.

ASYMPTOTES OF RATIONAL FUNCTIONS

**Vertical Asymptote -** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\*\*Solve by finding zeros of the denominator by setting the denominator equal to 0.\*\*

The vertical asymptotes are the x-values that are excluded from the domain.

$$f\left(x\right)=\frac{4}{x+2}$$

**Horizontal Asymptote -** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 Find the horizontal asymptotes by using the following rules of the degrees of $p(x)$ and $q(x)$.

Rule #1: If the numerator has the higher degree (exponent), there is NO horizontal asymptote.

$$f\left(x\right)=\frac{2x^{3}}{3x^{2}+1}$$

 If we divide out the degree, we can determine the end behavior of the graph.

Rule #2: If the denominator has the higher degree (exponent), $y=0$ is the horizontal

 asymptote.

$$f\left(x\right)=\frac{2x}{3x^{2}+1}$$

Rule #3: If the degree (exponent) is the same in both the numerator and denominator, then

the equation of the horizontal asymptote is the quotient of the leading coefficients of both the numerator and denominator.

$$f\left(x\right)=\frac{2x^{2}}{3x^{2}+1}$$

Find all vertical and horizontal asymptotes of the following functions.

1. $f\left(x\right)=\frac{2x^{2}+1}{x^{2}- 1 }$ b. $f\left(x\right)=\frac{x^{2}- x - 2}{x^{2}- x - 6}$

 c. $f\left(x\right)= \frac{4}{(x-3)^{3}}$ d. $f\left(x\right)=\frac{5x^{2}}{x+1}$

 e. $f\left(x\right)=\frac{6x}{x^{2}- 8x + 16}$ f. $f\left(x\right)=\frac{x^{3}}{x - 1}$